⇜Key Stage 4

Year 12

Year 13

University ˆ
Employment ⇝

GCSE Maths Grade 7 or above

Core Knowledge

Year 12 Autumn

Core Pure
Solve quadratic and cubic equations with real coefficients. Add, subtract, multiply and divide complex numbers. Understand and use the complex conjugate. Use and interpret Argand diagrams. Convert between the Cartesian form and the modulus-argument form of a complex number. Multiply and divide complex numbers in modulus argument form. Construct and interpret simple loci in the Argand diagram. Add, subtract and multiply conformable matrices. Multiply a matrix by a scalar.Understand and use zero and identity matrices. Use matrices to represent linear transformations in 2D. Successive transformations.
Find invariant points and lines for a linear transformation. Calculate determinants of 2×2 and 3×3 matrices and interpret as scale factors, including the effect on orientation Understand and use singular and non-singular matrices. Solve three linear simultaneous equations in three variables by use of the inverse matrix. Interpret geometrically the solution and failure of solution of three simultaneous linear equations.

Decision
The general ideas of algorithms and the implementation of an algorithm given by a flow chart or text. Bin packing, bubble sort and quick sort.
Use of the order of the nodes to determine whether a graph is Eulerian, semi-Eulerian or neither. Cost matrix reduction. Use of the Hungarian algorithm to find a least cost allocation. Modification of the Hungarian algorithm to deal with a maximum profit allocation. The minimum spanning tree (minimum connector) problem. Prim's and Kruskal's (greedy) algorithm. Dijkstra's algorithm for finding the shortest path. Cuts and their capacity. Use of the labelling procedure to augment a flow to determine the maximum flow in a network. Use of the max-flow min-cut theorem to prove that a flow is a maximum flow.

Year 12 Spring

Core Pure
Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. Understand and use the relationship between roots and coefficients of polynomial equations up to the quartic equations. Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). Construct proofs using mathematical induction. Contexts include sums of series, divisibility and powers of matrices. Understand and use the vector and Cartesian forms of the equation of a plane. Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane. Find the intersection between a line and a plane. Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.

Decision
Algorithm for finding the shortest route around a network (The Route Inspection Algorithm).
Formulation of problems as linear programs
Graphical solution of two variable problems using objective line and vertex methods including cases where integer solutions are required. Cuts and their capacity. Use of the labelling procedure to augment a flow to determine the maximum flow in a network. Use of the max-flow min-cut theorem to prove that a flow is a maximum flow. Two person zero-sum games and the pay-off matrix. Identification of play safe strategies and stable solutions (saddle points). Optimal mixed strategies for a game with no stable solution by use of graphical methods. Formulation of problems as linear programs Graphical solution of two variable problems using objective line and vertex methods including cases where integer solutions are required. Modelling of a project by an activity network, from a precedence table. Completion of the precedence table for a given activity network. Algorithm for finding the critical path. Construction of Gantt (cascade) charts.

Year 12 Summer

Core Pure
Derive formula for and calculate volumes of revolution. Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.
Find the distinct roots of and know that they form the vertices of a regular polygon in the Argand diagram. Use complex roots of unity to solve geometric problems.

Decision
Modelling of a project by an activity network, from a precedence table. Completion of the precedence table for a given activity network. Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities. Identification of critical activities and critical path(s). Calculation of the total float of an activity. Construction of Gantt (cascade) charts.
Use of recurrence relations to model appropriate problems. Solution of first and second order linear homogeneous and non-homogeneous recurrence relations. Use of recurrence relations to model appropriate problems. Solution of first and second order linear homogeneous and non-homogeneous recurrence relations. The planarity algorithm for planar graphs. Floyd's algorithm for finding the shortest path. The north-west corner method for finding an initial basic feasible solution. Use of the stepping-stone method for obtaining an improved solution. Improvement indices. Formulation of the transportation problem as a linear programming problem.

Year 13 Autumn

Core Pure
Understand the definitions of hyperbolic functions, including their domains and ranges, and be able to sketch their graphs. Differentiate and integrate hyperbolic functions. Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges. Derive and use the logarithmic forms of the inverse hyperbolic functions.
Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. Sketch curves, including use of trigonometric functions. Find the area enclosed by a polar curve. Understand and use the method of differences for summation of series including use of partial fractions. Find the Maclaurin series of a function including the general term. Recognise and use the Maclaurin series.

Decision
The practical and classical Travelling Salesman problems. The classical problem for complete graphs satisfying the triangle inequality. Determination of upper and lower bounds using minimum spanning tree methods. The nearest neighbour algorithm. Formulation of the Hungarian algorithm as a linear programming problem. Multiple sources and sinks. Vertices with restricted capacity. Determine the optimal flow rate in a network, subject to given constraints. The Simplex algorithm and tableau for maximising and minimising problems with ≤ constraints. The two-stage Simplex and big-M methods for maximising and minimising problems which may include both ≤ and ≥ constraints. Principles of dynamic programming. Bellman's principle of optimality. Stage variables and State variables. Use of tabulation to solve maximum, minimum, minimax or maximin problems.

Year 13 Spring

Core Pure
Derive formula for and calculate volumes of revolution. Evaluate improper integrals Understand and evaluate the mean value of a function. Integrate using partial fractions. Differentiate inverse trigonometric functions. Integrate functions and be able to choose trigonometric substitutions to integrate associated functions. Find and use an integrating factor to solve differential equations and recognise when it is appropriate to do so. Find both general and particular solutions to differential equations. Use differential equations in modelling in kinematics and in other contexts. Model damped oscillations using second order differential equations and interpret their solutions. Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models.

Decision
The Simplex algorithm and tableau for maximising and minimising problems with ≤ constraints. The two-stage Simplex and big-M methods for maximising and minimising problems which may include both ≤ and ≥ constraints. Use, construct and interpret simple decision trees. Use of expected monetary values (EMVs) and utility to compare alternative courses of action. Construct resource histograms (including resource levelling) based on the number of workers required to complete each activity. Scheduling the activities using the least number of workers required to complete the project. Reduction of pay-off matrices using dominance arguments. Optimal mixed strategies for a game with no stable solution by conversion of higher order games to linear programming problems that can then be solved by the Simplex algorithm.

Year 13 Summer

Core Pure
Find and use an integrating factor to solve differential equations and recognise when it is appropriate to do so. Find both general and particular solutions to differential equations. Use differential equations in modelling in kinematics and in other contexts.
Solve differential equations, by using the auxiliary equation. Solve differential equations, where \(a\) and \(b\) are constants, by solving the homogeneous case and adding a particular integral to the complementary function. Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation. Solve the equation for simple harmonic motion and relate the solution to the motion. Model damped oscillations using second order differential equations and interpret their solutions. Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models.

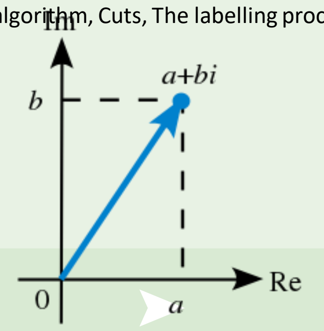
Decision
Construct resource histograms (including resource levelling) based on the number of workers required to complete each activity. Scheduling the activities using the least number of workers required to complete the project. Solution of first and second order linear homogeneous and non-homogeneous recurrence relations.

Concepts

Year 12 Autumn

Core Pure
Introduction of complex numbers, basic manipulation, Argand diagrams, Modulus and argument, Loci, Matrix addition, subtraction and multiplication, Inverse of 2×2 and 3×3 matrices, Simultaneous equations, Linear transformations, Complex conjugate, division and solving polynomial equations.

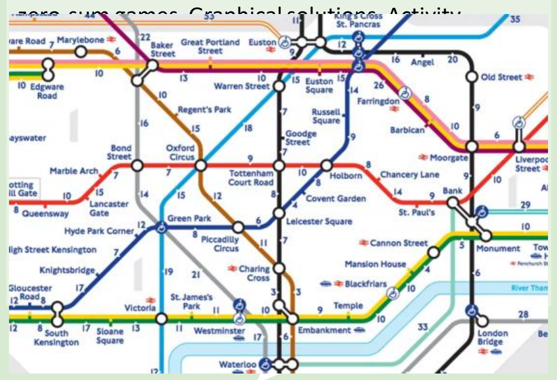
Decision
Introduction to algorithms, Sorting algorithms, Introduction to graph theory, The Hungarian Algorithm, Minimum connectors (spanning trees), Dijkstra's algorithm, Cuts, The labelling procedure.



Year 12 Spring

Core Pure
Sums of series, Roots of polynomial equations, Formulation of polynomial equation, Proof by mathematical induction, Vector and Cartesian equations of a line and a plane, Scalar product, Problems involving points, lines and planes

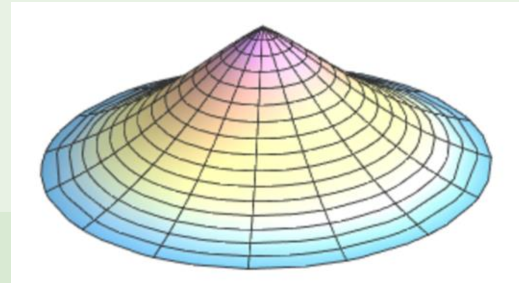
Decision
Route inspection problem, Formulation of problems, Maximum Flow-Minimum Cut Theorem, Two-person zero-sum games, Graphical solutions, Activity



Year 12 Summer

Core Pure
Volumes of revolution, Know and use z = re^iθ = r(cos θ + i sin θ), De Moivre's theorem, The nth roots of z = re^iθ and complex roots of unity.

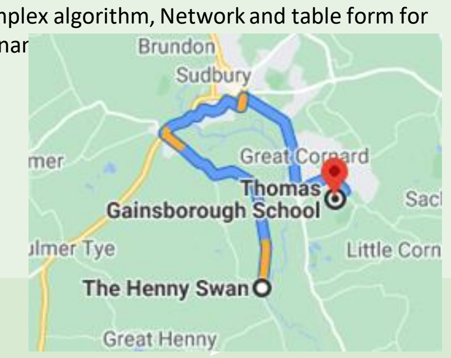
Decision
Critical path algorithm; earliest and latest event times, Total float; Gantt charts, Modelling using recurrence relations, Solving first order recurrence relations, Planarity algorithm, Floyd's algorithm, North-west corner method, Stepping stone method, Formulation as a linear programming problem.



Year 13 Autumn

Core Pure
sinh x, cosh x, tanh x and their inverses, Logarithmic forms of the inverse hyperbolic functions and integrate functions of the form (x^2 ± a^2)^-0.5, Convert between Cartesian and polar and sketch r(θ), Method of differences, Maclaurin series.

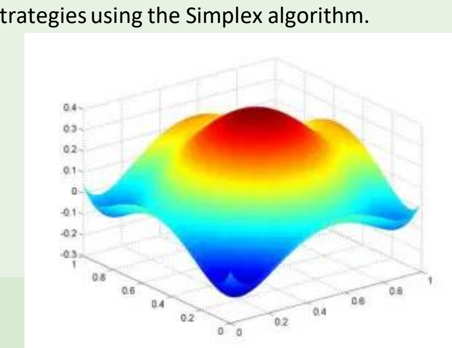
Decision
Travelling salesman problem, Formulation as a linear programming problem, Multiple sources and sinks, Optimal flow rates, Formulation of problems, Simplex algorithm, Network and table form for Dynar



Year 13 Spring

Core Pure
Improper integrals, Mean value of a function, Integrate using partial fractions, Differentiate inverse trigonometric functions and integrate using trigonometric substitutions, Volumes of revolution.

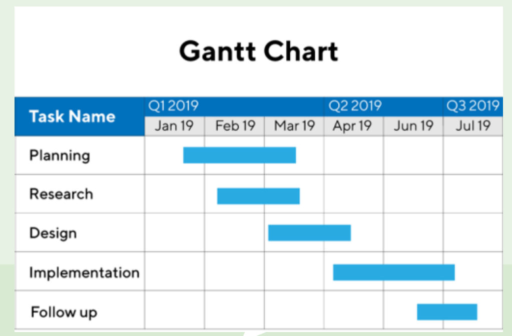
Decision
Big-M and two-stage Simplex, Decision trees, Expected monetary values (EMVs), Integrating factors to solve first order differential equations, Resource histograms, Dominance, Optimal mixed strategies using the Simplex algorithm.



Year 13 Summer

Core Pure
Second order differential equations of the form y'' + ay' + by = f(x), Modelling.

Decision
Scheduling, Solving second order recurrence relations.



Connections

Year 12 Autumn

Core Pure - Complex Numbers
Complex numbers are mainly used in electrical engineering techniques all the time, because Fourier transforms are used in understanding oscillations and wave behaviour that occur both in AC Current and in modulated signals.
Two-dimensional problems involving Laplace's equation (e.g. heat flow, fluid flow, electrostatics) are often solved using complex analysis, in particular conformal mapping
Complex analysis (transformation or mapping) is also used when we launch a satellite and here on earth we have z-plane but in space we have w-plane as well. So to study various factors we use transformation.

Year 12 Spring

Decision - Networks
The map of the London Underground is recognized across the world. It was designed by Harry Beck, an electrical draughtsman, who realised that, as the railway could not be seen, it was not necessary to put every station in its correct geographical location. He based the design on the electrical diagrams he was drawing as part of his job.

Year 12 Summer

Core Pure - Volumes of revolution
A solid of revolution is a three-dimensional object obtained by rotating a function in the plane about a line in the plane. The volume of this solid may be calculated by means of integration. Common methods for finding the volume are the disc method, the shell method, and Pappus's centroid theorem.

Volumes of revolution are useful for topics in engineering, medical imaging, and geometry. The manufacturing of machine parts and the creation of MRI images both require understanding of these solids.

Year 13 Autumn

Decision - Dynamic Programming
Dynamic Programming works on the principle of Optimality- If we have reached an optimal solution, then it's individual components must also be optimized

Real life examples
In Google Maps to find the shortest path between source and the series of destination (one by one) out of the various available paths.
In networking to transfer data from a sender to various receivers in a sequential manner.

Year 13 Spring

Core Pure - Differential Equations
Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines, from biology, economics, physics, chemistry and engineering. They can describe exponential growth and decay, the population growth of species or the change in investment return over time.

Year 13 Summer

Decision - Gantt Charts
A Gantt chart for construction projects is an easy approach to project management. It can be a great tool for contractors, homeowners, building companies, or government agencies.

Engineers from all over the world consider Gantt charts a simple and easy-to-use project management tool. They can easily group all construction stages in the columns, edit the tasks, and customize them to their needs.

Keeping important people in the loop is a key point in construction project management.

Vocabulary

Year 12 Autumn	Year 12 Spring	Year 12 Summer	Year 13 Autumn	Year 13 Spring	Year 13 Summer
Conjugate, real part, imaginary part, vector, magnitude, modulus, argument, radians, loci. Algorithm,iteration, bin packing, decreasing, vertices, nodes, edges,path,cycle,degree, valency, digraph, tree, spanning tree, k notation, isomorphic, planar.	capacity, source, sink, flow, feasibility condition, conservation condition, saturated, cut, initial flow, network, distance matrix, Conjugate, real part, imaginary part, complex conjugate, root, discriminant, Array, dimension, rows, columns, elements,	coefficient, quadratic, quartic, cubic, de Moivre, unity, exponential, multiple angle, Algorithm, flow chart, order, efficiency, loops, bubble sort, iteration, destination vertex, distance table, sequence table, entering cell, exiting cell, optimal solution, decision variables, objective function, constraints	Hyperbolic, sinh, cosh, tanh, domain, range, exponential, function, radical, Polar, Cartesian, Traversable, odd valency, cost matrix, row/column reduction, unbalanced problem, capacity, source, sink, flow, feasibility.	Improper, undefined, continuous, mean, integrate, partial, fraction, radical, inverse, decision variables, constraints, objective function, slack variables.	Integrating, factor, complementary, function, differential, equation, Activities, events, precedence table, activity networks, source node, sink node.

Assessment

Year 12 Autumn	Year 12 Spring	Year 12 Summer	Year 13 Autumn	Year 13 Spring	Year 13 Summer
Core Pure - Unit 1-3 Assessment Decision - Unit 1-2 Assessment	Core Pure - Unit 4-7 Assessment Decision - Unit 3-4 Assessment	Core Pure - Unit 8 Assessment Decision - Unit 5-6 Assessment	Core Pure - Unit Assessment Decision - Unit 7-8 Assessment	Core Pure - Unit Assessment Decision - Unit Assessment	Core Pure - Unit Assessment Decision - Unit 10-11 Assessment

University / Employment

Further Mathematical Qualifications
https://www.mathscareers.org.uk/article/degree-courses-a-level-mathematics/

Degree choices where A-level Mathematics is an essential requirement of nearly all universities
Actuarial Science, Aeronautical Engineering, Chemical Engineering, Civil Engineering, Economics, Electrical/Electronic Engineering, Engineering (General), Mathematics, Mechanical Engineering, Physics, Statistics.

Degree Choices where A-level Mathematics is an essential requirement by some, but not all universities
Accountancy, Chemistry, Computer Science, Management Studies.

Degree Choices where A-level Mathematics can make up one of an essential combination of subjects
Biochemistry, Biomedical Sciences (including Medical Science, Chemistry, Dentistry, Environmental Science/Studies, Geology/Earth Sciences, Materials Science, Medicine, Optometry (Opthalmic Optics), Pharmacy, Physiotherapy, Psychology, Sports Science/Physical, Teacher Training, Veterinary Science.

Degree choices where A-level Mathematics is listed as useful by most universities
Accountancy, Architecture, Biochemistry, Biology, Biomedical Sciences (including Medical Science), Business Studies, Chemistry, Computer Science, Dentistry, Dietetics, Geography, Law, Management Studies, Nursing and Midwifery, Orthoptics, Pharmacy, Philosophy, Physiotherapy, Planning, Psychology, Surveying, Teacher Training, Law, Materials Science, Mechanical Engineering, Medicine, Optometry (Opthalmic Optics), Physics, Veterinary Science.

Top Maths Universities
https://www.thecompleteuniversityguide.co.uk/league-tables/rankings/mathematics

1. University of Oxford
2. University of Cambridge
3. University of St Andrews
4. Durham University
5. Imperial College London
6. University of Warwick
7. University of Edinburgh
8. UCL (University College London)
9. Lancaster University
10. University of Bath

Preparing for an admissions test
STEP (Sixth Term Examination Paper)
Mathematics is a well-established mathematics examination designed to test candidates on questions that are similar in style to undergraduate mathematics.
STEP is used by the University of Cambridge and the University of Warwick. Other universities sometimes ask candidates to take STEP as part of their offer – in such cases, the university can advise on which papers to take.

Suggested reading
How to Study for a Mathematics Degree by Lara Alcock (ISBN 978-0-19-966132-9) explains what to expect at university and offers useful study advice.
Mathematics: A Very Short Introduction by Timothy Gowers (ISBN 978-0192853615) explains the differences between advanced maths and the maths learned at school.
Number: A Very Short Introduction by Peter M. Higgins (ISBN 978-0199584055) provides a comprehensive view of the idea of the number
Popular mathematics books by authors such as Simon Singh and Ian Stewart.
The University of Cambridge's recommended reading list .
Titles suggested by NRICH
Articles in Plus Magazine .

Further Mathematics

Excellence for each, for all

Thomas Gainsborough School